



FREQUENCY DISPERSION CHARACTERISTICS OF PHASE VELOCITIES IN SURFACE WAVE FOR ROTATIONAL COMPONENTS OF SEISMIC MOTION

H.-N. LI, L.-Y. SUN AND S.-Y. WANG

Department of Civil Engineering, Dalian University of Technology, Dalian 116024, People's Republic of China. E-mails: hnli@hybrid.mse.vt.edu, hongnanl@yahoo.com

(Received 11 July 2001, and in final form 20 March 2002)

When rotational components of ground motion produced by seismic surface waves are computed, the phase velocities must always be dealt with in earthquake engineering. In this paper, appropriate methods are presented to obtain the calculation formulas for the phase velocities of surface waves by applying the theory of elastic wave propagation. Frequency dispersion characteristics of phase velocities are discussed. The rocking component around a horizontal axis and the torsional component around a vertical axis, which are generated, respectively, by the Rayleigh and Love waves, are reasonably given. A procedure is developed to calculate the time histories of these rotational components.

© 2002 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

In the absence of having good records of rotational components of earthquake motion, the attempts have been made to define them in terms of the recorded translational components. Several studies [1–4] have shown the importance of rotational components in the seismic analysis and design of structures. It was shown [5] that during an earthquake, even symmetric structures could be expected to undergo substantial torsional excitation. The seismic design codes also prescribe “accidental eccentricity” in the design force calculations to explain the unknown torsional inputs and unintended eccentricity in the design of a building. However, no such provisions are made to account for the rocking components that can have a significant impact on the design of tall and rigid structures. One of the reasons why this input is not explicitly taken into account in the seismic design codes is the lack of reliable information on torsional ground spectra [6, 7]. Newmark [8] was perhaps the first researcher to establish a simple relationship between the torsional and translational components of a motion. It was based on the assumption of a constant velocity of wave propagation. Newmark’s idea was pursued further by other investigators who proposed procedures to obtain the seismic torsional response spectra from the time history records of horizontal components. Hart *et al.* [9] differentiated numerically two orthogonal translational records and obtained the associated free-field rotational motion. A similar technique was implemented by Nathan and MacKenzie [10] to generate the record of the torsional ground motion from the two components of the El Centro earthquake. When available, the acceleration time histories recorded by strong motion differential arrays can be used to generate rotational components by numerical differentiation of translational components. This technique was used by Niazi [11] to

estimate the torsional and rocking induced by differential displacements on long rigid foundations and by Oliveira and Bolt [12] to estimate the rotational components from five earthquakes recorded at the circular array in Taiwan, SMART-1.

The torsional ground motion response spectra have been developed by Tso and Hsu [13] and Rutenberg and Heidebrecht [14, 15]. The relation between the rotational power spectrum and the cross-correlation of vertical accelerations was studied by Castellani and Zembaty [16]. For the input motion considered by these investigators, these spectra had a flat peak in the periods of around 0.2–0.6 s. These periods' ranges have special significance for some high-frequency structures, such as medium height structures, stiff structures such as nuclear safety shell and structures with eccentric layouts likely to be affected by torsional base input.

Although it was used by several authors, the assumption of constant plane wave velocity of propagation made for calculating the rotational components time histories from the translational components is hard to justify. This velocity depends upon the frequency of the wave motion and the angle of incidence. More rational procedures have been developed by Trifunac [17], Lee and Trifunac [18, 19], Castellani and Boffi [20, 21], where the requirement of a constant plane wave velocity of propagation was relaxed and the dispersion and transient arrival times of waves in an elastic half-space were considered. However, these methods still assume that the angle of incidence is fixed and known. The angle of incidence also depends upon the frequencies of the impinging harmonics of the ground motion. For correct calculation of the rotational components from the corresponding translational components, this dependence of the angle of incidence as well as the velocity of propagation of the wave on the frequency of the harmonics constituting the ground motion at a site have been considered [22, 23].

Nevertheless, the studies mentioned above are only concerned with the cases of body waves of seismic motion. The surface waves (Rayleigh and Love waves) will be predominant in the composition of ground motion of earthquake when the site is relatively far field from seismic source. Although researchers [18, 19, 24] have explored the approach obtaining the rotational components from seismic surface waves, the effects of upper soil layers and frequency dispersion on the rotations have not been considered. In this paper, an appropriate method to include these effects is presented. The proposed procedure allows one to apply the contributions of the Rayleigh and Love waves to calculate time histories of the rotational component.

2. ROTATIONAL COMPONENTS FROM SURFACE WAVE

There are some wave functions satisfying the wave propagation equation and its boundary conditions except the body wave. These waves, which propagate along medium surfaces, are referred to as surface waves. In 1887, Lord Rayleigh mathematically derived the possibility of a combination of P and S waves that decreased in energy as they went deeper below the surface of an elastic half-space. It was later confirmed that the waves with characteristics Rayleigh had predicted did indeed exist among seismic waves. In 1911, A. E. H. Love predicted the existence of surface waves in mathematics whose motion was horizontal while the direction of their vibration was perpendicular to the direction of their propagation. The L phase in seismic waves approximated such characteristics and was called Love waves. Till date, researchers in engineering seismology have agreed that there are not only body wave (P and S waves) but also surface wave (Rayleigh and Love waves) in the components of seismic waves. In case of farfield source of earthquake, the surface waves are more probably dominant.

Assume that the medium of seismic wave propagation is homogeneous, isotropic and elastic half-space with multiple layers. The x -, y - and z -axis in coordinate system stand, respectively, for two horizontal axes and a vertical axis. In this section, the calculational formulations of seismic rotations obtained from the Rayleigh and Love waves are given according to the elastic wave propagation theory.

2.1. CASE OF RAYLEIGH WAVE

Figure 1 shows the case of Rayleigh wave propagation in soil layer, in which u , w and ϕ_{gy} are the horizontal, vertical and rocking components of earthquake motions, respectively, and ρ denotes the density of medium. Using Lamé potentials $\varphi(x, z, t)$ and $\psi(x, z, t)$, the Rayleigh waves in the top layer are given by

$$\varphi(x, z, t) = A \exp(-\sqrt{a^2 - k^2}z) \exp i(ax - \omega t), \tag{1a}$$

$$\psi(x, z, t) = B \exp(-\sqrt{a^2 - k^2}z) \exp i(ax - \omega t), \tag{1b}$$

where $k = \omega/\alpha$; $k' = \omega/\beta$; $a = \omega/V_R$; V_R is the phase velocity in the x direction; α and β are the velocities of P and S waves; and A and B are the amplitudes to be determined.

Thus, the two components of displacement can be expressed as

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial z} \tag{2a}$$

and

$$w = \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{2b}$$

Substituting equation (1) in equation (2), one may obtain

$$u = [Aia \exp(-\sqrt{a^2 - k^2}z) - B\sqrt{a^2 - k^2} \exp(-\sqrt{a^2 - k^2}z)] \exp i(ax - \omega t), \tag{3a}$$

$$w = -[A\sqrt{a^2 - k^2} \exp(-\sqrt{a^2 - k^2}z) + Bia \exp(-\sqrt{a^2 - k^2}z)] \exp i(ax - \omega t). \tag{3b}$$

The boundary condition of shear stress at the free surface is

$$\tau_{xz}|_{z=0} = \left[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right]_{z=0} = 0 \tag{4}$$

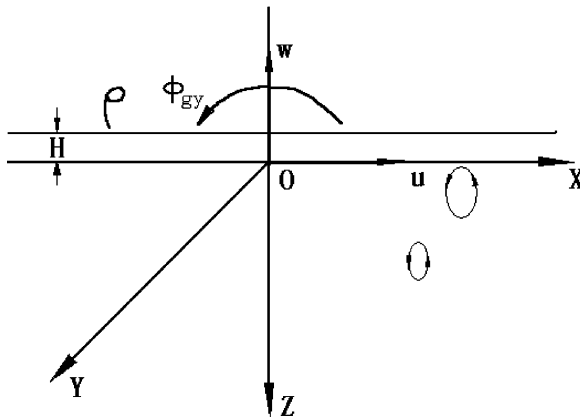


Figure 1. Rocking component from the Rayleigh wave.

according to the elastic theory [23], the rocking component around the y -axis is derived as follows:

$$\varphi_{gy} = \frac{1}{2} \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right) = \frac{\partial w}{\partial x}. \tag{5}$$

Furthermore, equation (5) using equation (3b) is expressed as

$$\begin{aligned} \varphi_{gy} &= \frac{\partial w}{\partial x} \\ &= -ia[A\sqrt{a^2 - k^2} \exp(\sqrt{a^2 - k^2}z) + Bia \exp(\sqrt{a^2 - k^2}z)] \exp i(ax - \omega t) \\ &= iaw = i \frac{\omega}{V_R} w. \end{aligned} \tag{6}$$

2.2. CASE OF LOVE WAVE

When there is a low-velocity wave soil layer above the elastic half-space, the Love waves may be generated in the topsoil layer and between two-layer media. These waves consist of SH-type waves trapped in each layer that propagate by multiple reflections within each layer. Actually, the Love waves may produce the seismic torsional component, φ_{gz} , around the vertical axis except that the rocking component around a horizontal axis comes from the Rayleigh wave. Figure 2 shows the case of the Love wave propagation in the soil layer, in which v is the horizontal component perpendicular to the wave propagation direction resulting from the Love wave.

Let V_{s1} and V_{s2} be shear wave velocities in the top and second soil layers with $V_{s1} > V_{s2}$, H be the depth of the top layer and $\lambda_1, \mu_1, \lambda_2, \mu_2$ be Lamé factors of the top and second layers. Thus, the Love waves in the two-layer media are expressed by

$$v_1(x, z, t) = [A \cos(pz) + B \sin(pz)] \exp i(ax - \omega t) \quad (-H < z < 0), \tag{7a}$$

$$v_2(x, z, t) = C \exp(-bz) \exp i(ax - \omega t) \quad (0 < z), \tag{7b}$$

where $p = \sqrt{k_1^2 - a^2}$, $b = \sqrt{a^2 - k_2^2}$, $k_1 = \sqrt{\omega^2/V_{s1}^2}$, $k_2 = \sqrt{\omega^2/V_{s2}^2}$, $a = \omega/V_L$, V_L is the phase velocity of Love wave, and A, B and C are the amplitudes to be determined.

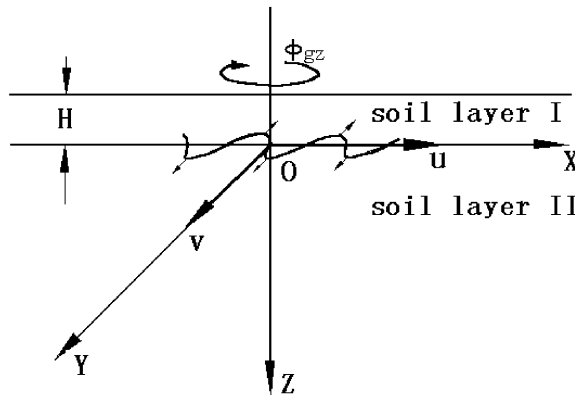


Figure 2. Torsional component from the Love wave.

The Love wave only produces the displacement in the y direction while displacements in the x and z directions are both zero if it propagates along the x direction due to the surface wave of SH type, i.e., $u = w = 0$. thus, the torsional component around vertical axis z is given by Li *et al.* [23].

$$\begin{aligned} \varphi_{gz} &= \frac{1}{2} \left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y} \right) \\ &= \frac{1}{2} \frac{\partial v_1}{\partial x} \\ &= \frac{1}{2} ia[A \cos(pz) + B \sin(pz)] \exp i(ax - \omega t) \\ &= \frac{1}{2} iav_1 = \frac{1}{2} \frac{i\omega}{V_L} v_1. \end{aligned} \tag{8}$$

Equations (6) and (8) give the relations between rotational components and translational components of seismic motion.

3. PHASE VELOCITIES WITH FREQUENCY DISPERSION

3.1. RAYLEIGH WAVE

Researchers [18, 19, 24] have investigated the propagating law of the Rayleigh wave at the surface of elastic half-space and presented the corresponding formulas without frequency dispersion. However, on the other side, it has been shown [25, 26] that the frequent dispersion comes into being as the Rayleigh wave travels in the non-homogeneous and layered media. Actually, the media in which the seismic wave propagates are not ideally elastic at all. The phase velocity of the Rayleigh wave varies with change of frequency in the topsoil layer above the elastic media.

Assume that there is a soil layer on the elastic half-space. Mass density of the layer is ρ . Due to the presence of soil layer, the boundary conditions become

$$\sigma_{zz}|_{z=0} = \left(\lambda\theta + 2\mu \frac{\partial w}{\partial z} \right) \Big|_{z=0} = \rho \frac{\partial^2 w}{\partial t^2}, \tag{9}$$

$$\tau_{zx}|_{z=0} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \Big|_{z=0} = 0, \tag{10}$$

where λ and μ is the Lamé factors; and $\theta = \partial u/\partial x + \partial v/\partial y + \partial w/\partial z$.

Let

$$K = \frac{\alpha}{\beta} = \sqrt{\frac{\lambda + 2\mu}{\mu}}. \tag{11}$$

Substituting equation (11) into equation (9), one obtains

$$\sigma_{zz}|_{z=0} = \mu K^2 \frac{\partial w}{\partial z} + \mu(K^2 - 2) \frac{\partial u}{\partial x} = \rho \frac{\partial^2 w}{\partial t^2}. \tag{12}$$

Thus, equations (10) and (12) compose new stress boundary conditions. Thereafter, substituting equation (3b) in two sides of the normal stress condition equation (12) at the

free surface ($z = 0$):

$$\begin{aligned} \sigma_{zz}|_{z=0} &= \{-\mu K^2[-\sqrt{a^2 - k^2}A\sqrt{a^2 - k^2} \exp(-\sqrt{a^2 - k^2})z \\ &\quad - Bia\sqrt{a^2 - k'^2} \exp(-\sqrt{a^2 - k'^2})z] \exp i(ax - \omega t) \\ &\quad + \mu ia(K^2 - 2)[Aia \exp(-\sqrt{a^2 - k^2})z \\ &\quad - B\sqrt{a^2 - k'^2} \exp(-\sqrt{a^2 - k'^2})z] \exp i(ax - \omega t)\}|_{z=0} \\ &= \{-\mu K^2[-(a^2 - k^2)A - Bia\sqrt{a^2 - k'^2}] \\ &\quad + \mu ia(K^2 - 2)[Aia - B\sqrt{a^2 - k'^2}]\} \exp i(ax - \omega t) \end{aligned} \tag{13}$$

and

$$\begin{aligned} \rho \frac{\partial^2 w}{\partial t^2} \Big|_{z=0} &= \{\rho \omega^2[A\sqrt{a^2 - k^2} \exp(-\sqrt{a^2 - k^2})z + Bia \exp(-\sqrt{a^2 - k'^2})z] \exp i(ax - \omega t)\}|_{z=0} \\ &= \rho \omega^2[A\sqrt{a^2 - k^2} + Bia] \exp i(ax - \omega t), \end{aligned} \tag{14}$$

one can obtain

$$(2\mu a^2 - \mu K^2 k^2 - \rho \omega^2 \sqrt{a^2 - k^2})A + (2\mu \sqrt{a^2 - k'^2} - \rho \omega^2)aBi = 0. \tag{15}$$

Similarly, substitution of equation (3) in the shear stress condition equation (10) at the free surface ($z = 0$) gives

$$\begin{aligned} \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \Big|_{z=0} &= \{\mu[Aia(-\sqrt{a^2 - k^2}) \exp(-\sqrt{a^2 - k^2})z \\ &\quad + B\sqrt{a^2 - k'^2} \sqrt{a^2 - k'^2} \exp(-\sqrt{a^2 - k'^2})z] \exp i(ax - \omega t) \\ &\quad - ia[A\sqrt{a^2 - k^2} \exp(-\sqrt{a^2 - k^2})z + Bia \exp(-\sqrt{a^2 - k'^2})z] \exp i(ax - \omega t)\}|_{z=0} \\ &= \mu[-2Aia\sqrt{a^2 - k^2} + (2a^2 - k'^2)B] \exp i(ax - \omega t) \\ &= 0. \end{aligned} \tag{16}$$

After simplifying, equation (16) reduces to

$$-2ia\sqrt{a^2 - k^2}A + (2a^2 - k'^2)B = 0. \tag{17}$$

If their solutions are true and non-zero for arbitrary values of coefficients A and B in combination of equations (15) and (17), the determinant of their coefficients must be equal to zero:

$$\begin{vmatrix} 2\mu a^2 - \mu K^2 k^2 - \rho \omega^2 \sqrt{a^2 - k^2} & i(2\mu \sqrt{a^2 - k'^2} - \rho \omega^2)a \\ -2ia\sqrt{a^2 - k^2} & 2a^2 - k'^2 \end{vmatrix} = 0, \tag{18}$$

which can be expanded to

$$(2\mu a^2 - \mu k'^2 - \rho \omega^2 \sqrt{a^2 - k^2})(2a^2 - k'^2) - 2a^2 \sqrt{a^2 - k^2}(2\mu \sqrt{a^2 - k'^2} - \rho \omega^2) = 0, \tag{19}$$

Let $k = k_1$, $k' = k_2$, $k'/k = k_2/k_1 = \sqrt{(\lambda + 2\mu)/\mu} = K$, then $k_2 = Kk_1$. Equation (19) is rewritten as follows:

$$4\mu a^4 - 4\mu a^2 \frac{\omega^2}{\beta^2} + \mu \frac{\omega^4}{\beta^4} + \frac{\gamma \omega^4}{\beta^2} \sqrt{a^2 - \frac{\omega^2}{\alpha^2}} - 4\mu a^2 \sqrt{a^2 - \frac{\omega^2}{\alpha^2}} \sqrt{a^2 - \frac{\omega^2}{\beta^2}} = 0. \tag{20}$$

The factor a can be solved from equation (20). It is known that a will be a function of frequency ω . And then the phase velocity, $V_R = \omega/a$, will also be a function of frequency

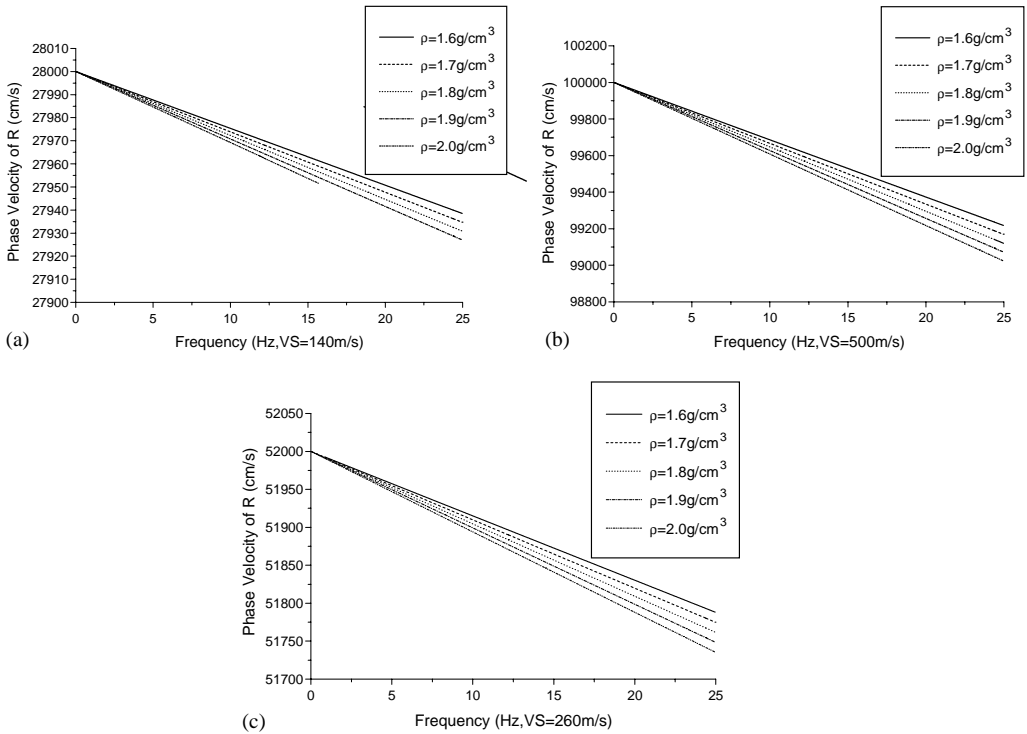


Figure 3. Phase velocity of Rayleigh wave versus frequency.

ω . Figure 3 shows the variation of phase velocity of the Rayleigh waves with the frequency, in which V_s is shear wave velocity in the upper soil layer and chosen as 140, 260 and 500 m/s, respectively, standing for soft, medium and solid soils. It can be seen from these figures that the V_R decreases almost linearly with the increase of frequency and decreases faster when the soil density, ρ , is larger.

3.2. LOVE WAVE

The stress and displacement boundary conditions should be continuous due to the existence of the Love waves between two-layer media and the stress boundary condition at free surface is zero, i.e.,

$$\text{Stress continuum : } \mu_1 \frac{\partial v_1}{\partial z} \Big|_{z=0} = \mu_2 \frac{\partial v_2}{\partial z} \Big|_{z=0}, \tag{21}$$

$$\text{Displacement continuum : } v_1 \Big|_{z=0} = v_2 \Big|_{z=0}. \tag{22}$$

$$\text{Free surface : } \mu_1 \frac{\partial v_1}{\partial z} \Big|_{z=-D} = 0. \tag{23}$$

Substitution of equation (7) into equation (21) gives

$$\begin{aligned} \mu_1 \frac{\partial v_1}{\partial z} &= \mu_1 [-Ap \sin(pz) + Bp \cos(pz)] \exp i(ax - \omega t) |_{z=0} \\ &= \mu_1 p B \exp i(ax - \omega t) = \mu_2 \frac{\partial v_2}{\partial z} \\ &= \mu_2 (-b) C \exp(-bz) \exp i(ax - \omega t) |_{z=0} \\ &= -b C \mu_2 \exp i(ax - \omega t). \end{aligned} \tag{24}$$

Therefore,

$$\mu_1 p B \exp i(ax - \omega t) = -b C \mu_2 \exp i(ax - \omega t). \tag{25}$$

By simplification, the equation (25) reduces to

$$\mu_1 p B + b C \mu_2 = 0. \tag{26}$$

Using equation (22) with equation (7) gives

$$[A \cos(pz) + B \sin(pz)] \exp i(ax - \omega t) |_{z=0} = C \exp(-bz) \exp i(ax - \omega t) |_{z=0} \tag{27}$$

so that

$$A - C = 0. \tag{28}$$

Furthermore, substitution of equation (7a) in the stress boundary condition equation (23) yields

$$A \sin(Dp) + B \cos(Dp) = 0, \tag{29}$$

from

$$\mu_1 \frac{\partial v_1}{\partial z} \Big|_{z=-D} = \mu_1 [-Ap \sin(pz) + Bp \cos(pz)] \exp i(ax - \omega t) |_{z=-D} = 0. \tag{30}$$

By combination of equations (26), (28) and (29), one can obtain

$$\mu_1 p \sin(Dp) = \mu_2 b \cos(Dp). \tag{31}$$

By use of the definitions of p and b , equation (31) is expressed as

$$\mu_1 \sqrt{\frac{1}{V_{S1}^2} - \frac{1}{V_L^2}} \sin \left(D\omega \sqrt{\frac{1}{V_{S1}^2} - \frac{1}{V_L^2}} \right) - \mu_2 \sqrt{\frac{1}{V_L^2} - \frac{1}{V_{S2}^2}} \cos \left(D\omega \sqrt{\frac{1}{V_{S1}^2} - \frac{1}{V_L^2}} \right) = 0. \tag{32}$$

Thus, the phase velocity, V_L , of the Love wave with the variation of frequency can be obtained from equation (32). Figure 4 presents the phase velocity of the Love waves versus the frequency, in which the shear wave velocity, V_{s1} , of the first soil layer is always selected to be less than the shear wave velocity, V_{s2} , of the second soil layer. It can be noted that:

- (1) the phase velocity, V_L , reduces non-linearly with the increase of frequency, but tends towards a constant in all cases when the frequency is more than a certain value;
- (2) the larger the difference between the shear wave velocities of first and second soil layers is, the faster the phase velocity, V_L , drops;
- (3) the thicker the upper (first) soil layer is, the faster the phase velocity, V_L , decreases according to the Figure 4.

4. NUMERICAL RESULTS

For illustrations, a set of three translational components of seismic records measured from Landers Earthquake on June 28, 1992, which is farfield source of earthquake, with

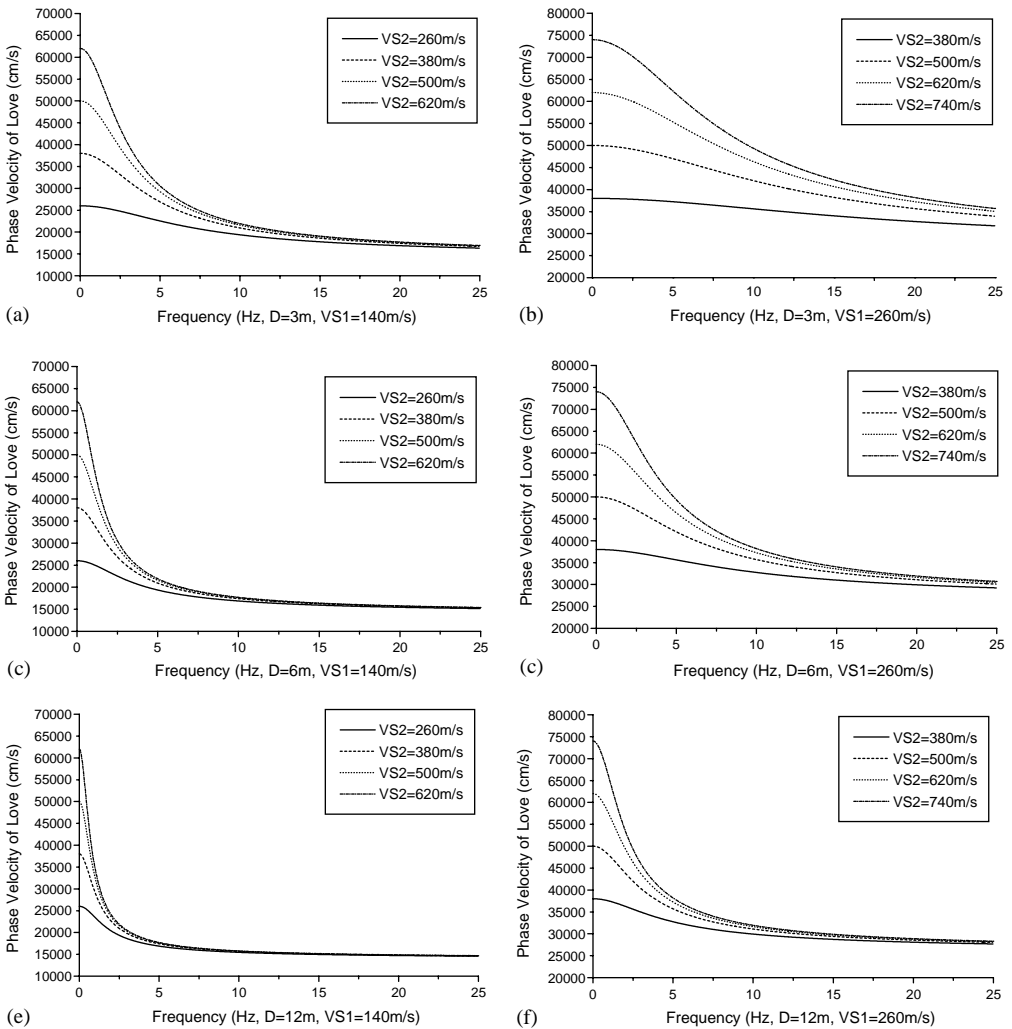


Figure 4. Phase velocity of Love wave versus frequency.

an epicentral distance of more than 100 km, has been used to obtain the time histories for the corresponding rotational components. They are approximately regarded as predominant composition with the surface waves, i.e., Rayleigh and Love waves. The time histories of the three translational components and their respective power spectra are shown in Figures 5 and 6, in which the NS Comp., EW Comp. and UD Comp. represent the north–south, east–west and up–down components of ground motion respectively. The calculation procedure to obtain the rotational components is proposed in the following:

The time histories and their corresponding power spectra of rocking and torsional components, obtained by the above process, are shown in Figures 7 and 8. During the calculation of rocking component, the factor, a , is firstly solved from equation (20) according to the selected seismic record and then the phase velocity, $V_R = \omega/a$, is obtained. Thus, the rocking component can be obtained by substituting V_R in equation (6). On the computation of torsional component, the first thing is to calculate the phase velocity, V_L , from equation (32). And then the torsional component is obtained by

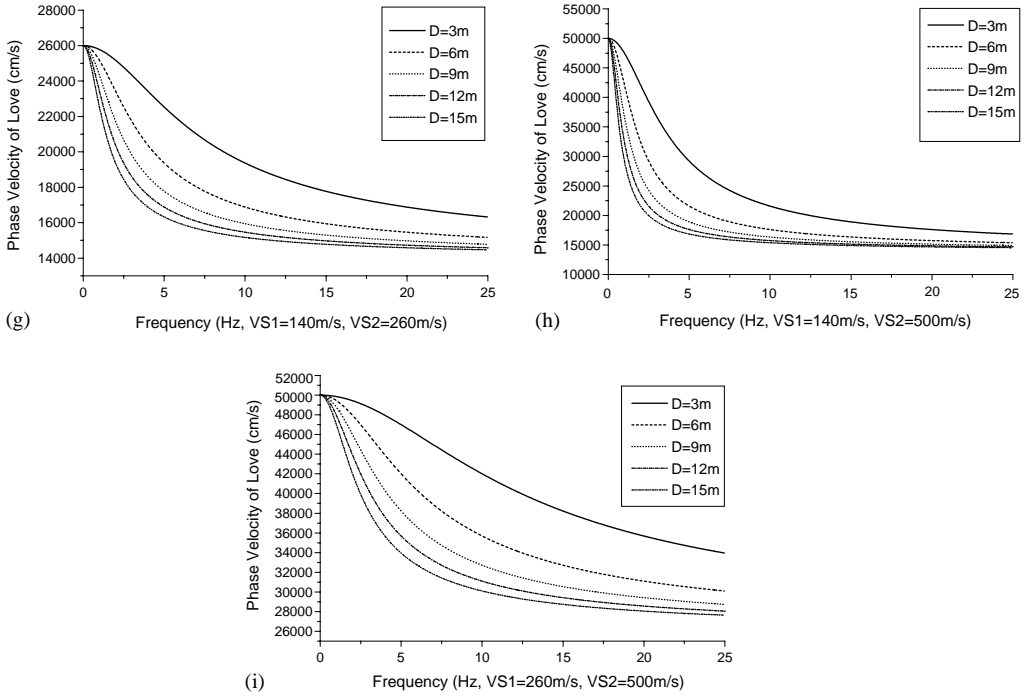


Figure 4. Continued.

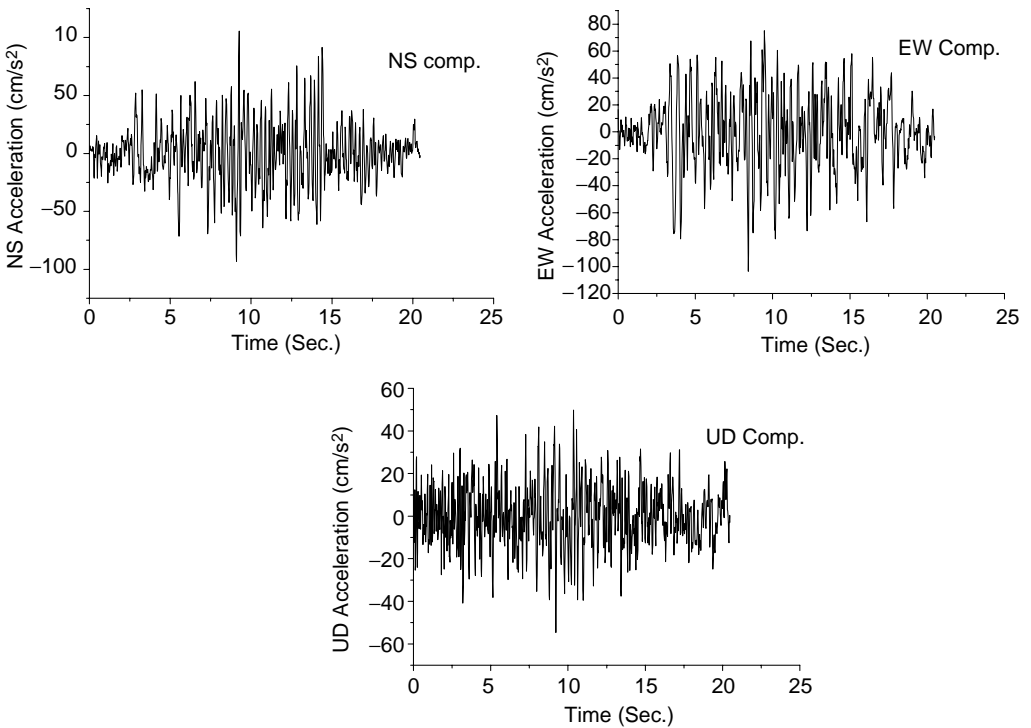


Figure 5. Time histories of translational accelerations (Landers No. 32075).

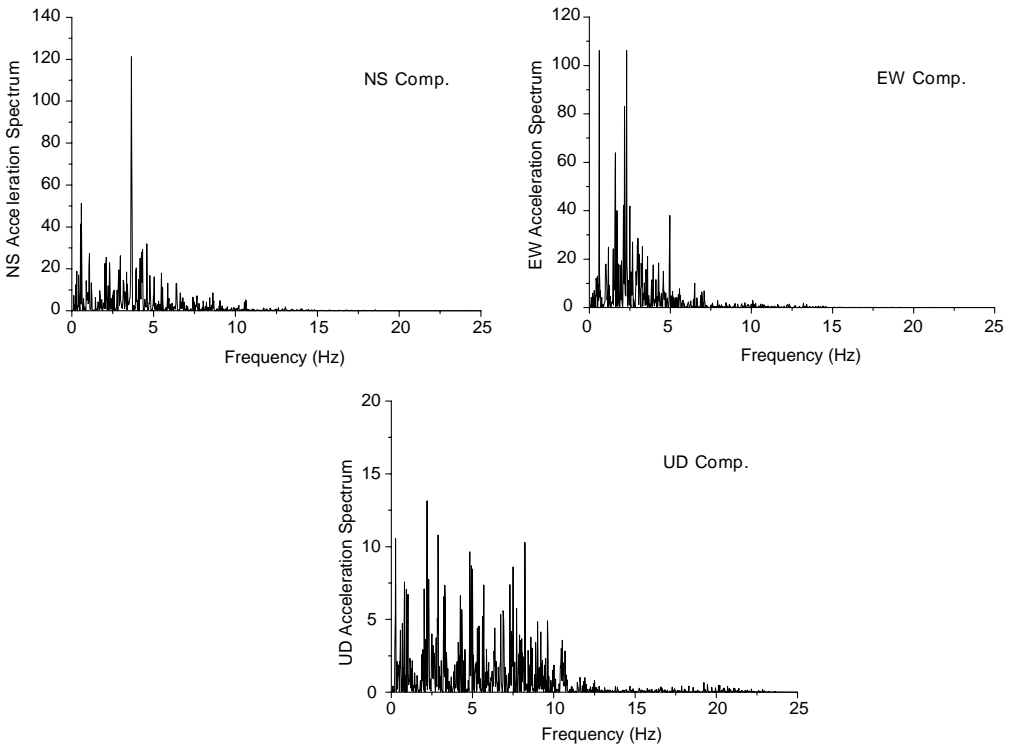


Figure 6. Power spectra of translational accelerations (Landers No. 32075).

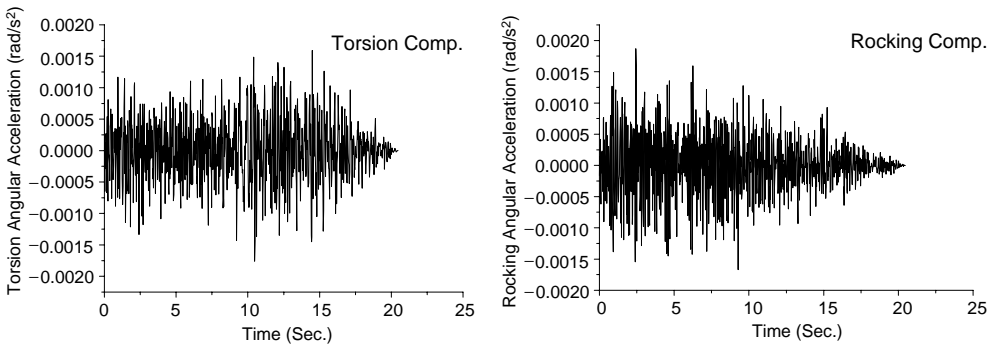


Figure 7. Time histories of rotational accelerations (Landers No. 32075).

substituting V_L in equation (8). The power spectrum represents the energy distribution of a time history. The comparison of these rotational component spectra with the spectra of translational components has shown that rotational spectra have more energy in the high-frequency range because the power spectral values of two horizontal components are almost equal to zero at the frequencies more than 8 Hz and the power spectral values of vertical component is very little at the frequencies more than 10 Hz, while the rotational spectra are still rich even more than 10 Hz. These also show that the rotational spectra attenuate slower than the translational spectra as the spectral components.

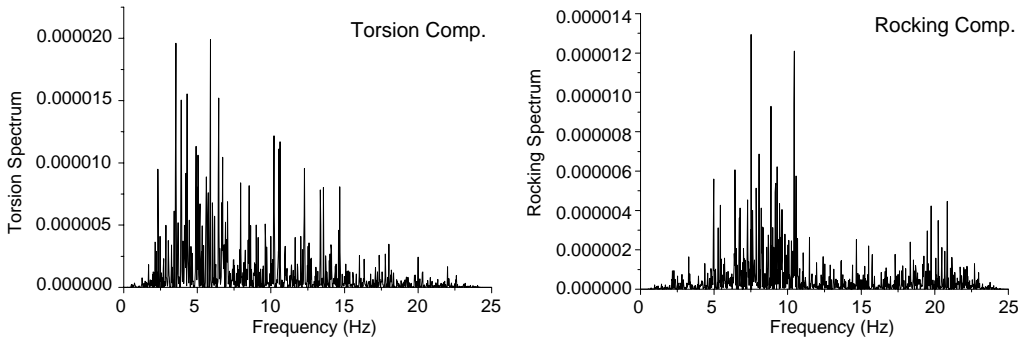


Figure 8. Power spectra of rotational accelerations (Landers No. 32075).

5. CONCLUSION

The need for providing the resistance to torsion in buildings subjected to seismic ground motion, in addition to the computed effects of asymmetry, has been traditionally recognized by code provisions through the stipulation of the so-called “accidental eccentricity”, which is typically taken as 5% of the bigger plan dimension of the building. This judgment actually accounted for two portions: the discrepancies between the computed and as-built stiffness features of lateral load-resisting members and seismic rotational motions. As the rotational records have not as yet been measured directly for engineering application, the reasonable methods [22, 23] have been given to make it possible for the estimation of this effect on buildings to the rotational inputs of earthquake caused by the arrival of body waves in the case of nearfield source.

In this paper, the appropriate methods are presented to obtain the rotational components caused by the arrival of the surface waves, i.e., the Rayleigh and Love waves, in the case of relative farfield seismic ground motions, which continues the authors’ previous work. The rocking component around a horizontal axis and the torsional component around a vertical axis are generated, respectively, by the Rayleigh and Love waves. At the same time, the calculation formulations of phase velocities about these waves with frequency dispersion are derived. A procedure is developed to compute the time histories of seismic rotational motions. Furthermore, numerical results have shown that the rotational motions have more energy than the translational motions in high-frequency range.

ACKNOWLEDGMENTS

The research in this paper is supported by the Outstanding Youth Science Foundation of the National Science Foundation of China. This support is greatly acknowledged.

REFERENCES

1. J. BIELAK 1978 *Earthquake Engineering and Structural Dynamics* **6**, 17–30. Dynamic response of nonlinear building–foundation system.
2. A. M. ABDEL-GHAFFAR and L. I. RUBIN 1984 *American Society of Civil Engineers Journal of Engineering Mechanics* **110**, 1467–1484. Torsional earthquake response of suspension bridges.

3. R. K. GOEL and A. K. CHOPRA 1994 *American Society of Civil Engineers Journal of Structural Engineering* **120**, 161–179. Dual-level approach for seismic design of asymmetric-plan buildings.
4. M. TAKEO and M. H. ITO 1997 *Geophysical Journal International* **129**, 319–329. What can be learned from rotational motions excited by earthquakes.
5. A. M. AWAD and J. L. HUMAR 1984 *Canadian Journal of Civil Engineering* **11**, 48–56. Dynamic response of buildings to ground rotational motion.
6. R. L. NIGBOR 1994 *Bulletin of Seismological Society for America* **84**, 1665–1669. Six-degree-of-freedom ground-motion measurement.
7. M. TAKEO 1998 *Geophysical Research Letters* **25**, 789–792. Ground rotational motions recorded in near-source region of earthquakes.
8. N. M. NEWMARK 1969 *Proceedings of the 4th World Conference on Earthquake Engineering, Santiago, Chile* Vol. 2, A3. 19–A3. 32. Torsion in symmetrical buildings.
9. G. C. HART, R. M. DIJULIO and M. LEW 1975 *American Society of Civil Engineers Journal of the Structural Division* **101**, 397–415. Torsional response of high-rise buildings
10. N. D. NATHAN and J. R. MACKENZIE 1975 *Canadian Journal of Civil Engineering* **2**, 430–436. Rotational components of earthquake motions.
11. M. NIAZI 1986 *Earthquake Engineering and Structural Dynamics* **14**, 531–542. Inferred displacements, velocities and rotations of a long rigid foundation located at El Centro differential array site during the 1979 imperial valley, California earthquake.
12. C. S. OLIVERA and B. A. BOLT 1989 *Earthquake Engineering and Structural Dynamics* **18**, 517–526. Rotational components of surface strong ground motion.
13. W. K. TSO and T.-I. HSU 1977 *Earthquake Engineering and Structural Dynamics* **6**, 375–382. Torsional spectrum for earthquake motions.
14. A. RUTENBERG and A. C. HEIDBRECHT 1985 *Canadian Journal of Civil Engineering* **12**, 583–592. Rotational ground motion and seismic codes.
15. A. RUTENBERG and A. C. HEIDBRECHT 1985 *Earthquake Engineering and Structural Dynamics* **13**, 543–557. Response spectra for torsion, rocking and rigid.
16. A. CASTELLANI and G. BOFFI 1986 *Earthquake Engineering and Structural Dynamics* **14**, 751–767. Rotational components of the surface ground motion during an earthquake.
17. M. D. TRIFUNAC 1982 *Soil Dynamics and Earthquake Engineering* **1**, 11–19. A note on rotational components of earthquake motions on ground surface for incident body waves.
18. V. W. LEE and M. D. TRIFUNAC 1985 *Soil Dynamics and Earthquake Engineering* **4**, 132–139. Torsional accelerograms.
19. V. W. LEE and M. D. TRIFUNAC 1985 *Soil Dynamics and Earthquake Engineering* **6**, 75–89. Rocking strong earthquake accelerations.
20. A. CASTELLANI and G. BOFFI 1989 *Earthquake Engineering and Structural Dynamics* **18**, 785–797. On the rotational components of seismic motion.
21. A. CASTELLANI and Z. ZEMBATY 1996 *Engineering Structure*, **18**, 597–602. Comparison between earthquake rotation spectra obtained by different experimental sources.
22. L. E. SUAREZ, H. N. LI and M. P. SINGH 1994 *Proceedings of the 5th National Conference on Earthquake Engineering, Chicago, U.S.A.*, Vol. III, 179–188. Rotational components of earthquake motions.
23. H. N. LI, L. E. SUAREZ and M. P. SINGH 1997 *Earthquake Engineering and Engineering Vibration* **17**, 37–50. Rotational components of earthquake ground motions.
24. M. A. BRAVO and F. J. SANCHEZ-SESMA 1990 *Soil Dynamics and Earthquake Engineering* **9**, 132–139. Seismic response of alluvial valleys for incident P, SV and Rayleigh waves.
25. Y. M. CHEN and S. M. WU 1991 *Journal of Zhejiang University* **25**, 40–52. Solution approach for characterized equation of Rayleigh wave in layered soil.
26. T. D. XIA and Y. Q. CAI 1996 *China Journal of Geotechnical Engineering* **18**, 28–34. Further investigation for calculating dynamic response using features of Rayleigh surface.